CONVECTIVE INSTABILITY OF A MIXTURE WITH CONCENTRATION HEAT SOURCES

A. K. Kolesnikov and V. I. Yakushin

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We investigate the convective stability of a horizontal layer of a two-component binary mixture with internal heat release whose intensity depends on the concentration of one of the components. We present curves of neutral stability and graphs of the amplitudes of critical perturbations.

We consider a binary mixture, one of the components of which causes heat release. Internal sources of heat depending on the concentration ("concentration" sources of heat) can arise in a mixture as a result of processes of radioactive decay, selective absorption of light, or exothermic chemical reaction of nonzero order. An example of a mixture with concentration heat sources of radioactive type is the asthenospheric layer of the mantle of the earth [1]. Internal sources with intensity depending on the concentration also arise for the propagation of radiation in a layer with an impurity having large light absorption [2]. In this case the energy absorbed by an impurity can be converted into internal degrees of freedom, as a result of which there is rapid local heating near the impurity. Finally the model of a mixture with concentration heat sources gives a good description of certain types of exothermic chemical processes, operating with large thermal effects in a strongly diluted reagent.

Conditions of formation of convection in such system should be noticeably different from those for an ordinary nonisothermal binary mixture. The difference is connected first of all with the possibility of diffusion redistribution of heat sources.

We investigate the convective stability of an incompressible binary mixture with concentration heat sources. The mixture fills an infinite horizontal layer bounded by parallel isothermal planes z = 0 and z = d. On the lower boundary of the layer there is a constant concentration of heat-releasing component $C = C^{(0)}$; on the upper boundary C = 0. We assume that the density of the mixture depends linearly on the temperature and concentration

$$\rho = \rho_0 (1 - \beta_1 T - \beta_2 C),$$

where β_1 is the ordinary coefficient of thermal expansion, and $\beta_2 = -1/\rho_0(\partial\rho/\partial C)_{T,p}$ determines the dependence of the density on concentration. For a light active component $\beta_2 > 0$; if the heat release is due to the heavy component, then $\beta_2 < 0$.

The system of equations describing the thermal-concentration convection in a binary incompressible mixture includes the equation of motion, the heat equation and diffusion equation, and the equation of continuity [3]. The presence of concentration sources of heat leads to the appearance in the heat equation of the additional term

$$\frac{Q}{\rho_0 c_p} C, \tag{1}$$

which depends linearly on the concentration of the active component, which corresponds, for example, to the exothermic reaction of first order. In expression (1), Q is the specific intensity of heat release and c_p is the specific heat.

With account of (1) the equations of convection in the binary mixture in dimensionless variables, assuming that the Boussinesq approximation is valid and that there is no thermal diffusion or diffusion heat conduction, take the form

$$\frac{\partial \mathbf{v}}{\partial t} + \frac{1}{P}(\mathbf{v}\nabla)\mathbf{v} = -\nabla p + \Delta \mathbf{v} + (RT + R_d C)\boldsymbol{\gamma},$$

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$$P\frac{\partial T}{\partial t}+\mathbf{v}\nabla T=\Delta T+6C,$$

$$P_d\frac{\partial C}{\partial t}+\frac{P_d}{P}\mathbf{v}\nabla C=\Delta C,\ \operatorname{div}\mathbf{v}=0. \tag{2}$$

In system (2) we use the ordinary notation (C is the dimensionless concentration of the heat-releasing component; the unit vector γ is directed vertically upward). As the units of measurement of distance, time, velocity, temperature, concentration, and pressure we choose the quantities d, d^2/ν, χ/d, qd^2, $C^{(0)}$, and $\rho_0\nu\chi/d^2$, where $q = QC^{(0)}/6\rho c_p\chi$.

The equations contain four dimensionless parameters: $R = g\beta_1 qd^5/\nu\chi$ is the Rayleigh number; $R_d = g\beta_2 C^{(0)}d^3/\nu\chi$, its concentration analog (the diffusion Rayleigh number); $P = \nu/\chi$, the Prandtl number, and $P_d = \nu/D$, the Schmidt number (the diffusion Prandtl number).

The boundaries of the layer are assumed to be rigid and are maintained at the same temperature assumed at the reference origin. On the lower boundary, as has already been noted, we are given a constant concentration C = 1; on the upper boundary the active component is absent. Thus, the velocity, temperature, and concentration satisfy the following boundary conditions:

$$\begin{aligned}&\text{for}\quad z=0\ \ \mathbf{v}=0,\ T=0,\ C=1;\\&\text{for}\quad z=1\ \ \mathbf{v}=0,\ T=0,\ C=0.\end{aligned} \tag{3}$$

The boundary problem (2), (3) that has been formulated has a stationary solution corresponding to mechanical equilibrium:

$$\mathbf{v}_0=0,\ T_0=z(z^2-3z+2),\ C_0=1-z. \tag{4}$$

From the form of the equilibrium profiles of the temperature and concentration (4) it follows that in the layer there are regions with potentially unstable stratification of density, which is due to the temperature distribution, and in the case of a light active component this is also due to the distribution of concentration.

We investigate the stability of the distributions (4) with respect to the onset of convection. In order to do this we consider the behavior of the small normal perturbations $\sim\exp[-\lambda t + i(k_1x + k_2y)]$, where $\lambda = \lambda_r + i\lambda_i$; λ_r is the real part and λ_i is the imaginary part of the decrement λ.

After linearization of the initial system (2) with respect to the small perturbations of velocity, of temperature, and of concentration, and elimination of the pressure, we obtain for their amplitudes w(z), θ(z), η(z), a system of ordinary homogeneous differential equations

$$\begin{aligned}-\lambda(w''-k^2w)&=(w^{\mathrm{IV}}-2k^2w''+k^4w)-Rk^2\theta-R_dk^2\eta,\\-\lambda P\theta&=(\theta''-k^2\theta)+6\eta-wT_0',\\-\lambda P_d\eta&=(\eta''-k^2\eta)-\frac{P_d}{P}wC_0'.\end{aligned} \tag{5}$$

Here $k^2 = k_1^2 + k_2^2$.

The boundary conditions for w, θ, and η in accordance with (3) have the form

$$w=w'=\theta=\eta=0\quad\text{for}\quad z=0;\ 1. \tag{6}$$

For P_d = P and R = 0 the problem (5), (6) is turned into a concentration analog of the known Rayleigh problem. For the case P_d = 0, Eqs. (5) with boundary conditions (6) describe the formation of convection in a layer with linearly distributed internal sources of heat [4]. For fixed concentration of the active component (C_0 = const) for P_d = 0, the problem (5), (6) reduces to the problem of the stability of a liquid with homogeneous heat release, which was considered in [5] for a thermally insulated lower boundary.

The decrements λ(P, P_d, R, R_d, k) are eigenvalues of the spectral problem (5), (6), and the amplitudes of the perturbations are its eigenfunctions.

For the solution, the system of equations for the complex amplitudes w, θ, and η was reduced to a system of 16 real first-order equations for the real and imaginary parts of the functions w, w', w'', w''', θ, θ'', η, and η'. The Runge–Kutta–Merson method [6] was used to construct four linearly independent particular

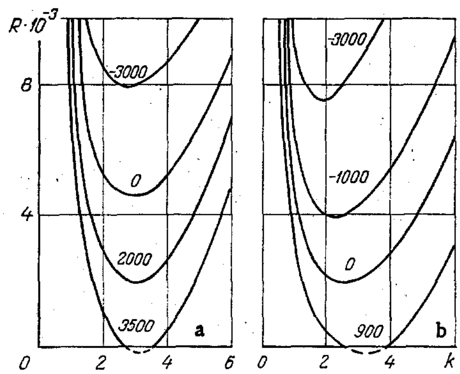

Fig. 1. Curves of neutral stability for various values of R_d: a) $L = 0.5$; b) 2.0.

solutions, satisfying the boundary conditions at the initial point of integration. The requirements of the existence of a nontrivial solution of the problem and satisfaction of the boundary conditions at the final point of the interval of integration lead to a characteristic relation, determining both parts of the complex decrement λ. A fundamental result of the calculations is the finding of the spectrum of the decrements as a function of all the parameters. The values $\lambda_r > 0$ correspond to damping of the perturbations, and $\lambda_r < 0$ correspond to growth; the stability boundary is determined by the condition $\lambda_r = 0$.

The modification of the Runge–Kutta method that is used enables us to effectively carry out a step-by-step verification of the accuracy of the integration of the equations. As a control example for $P_d = P$ and $R = 0$ we determined the minimum critical quantity R_{d_*} for the concentration Rayleigh problem, the value of which $R_{d*} = 1707.762$ completely coincides with that given in [3].

We turn to a discussion of the results obtained.

It is known that in binary systems under specific conditions it is possible to have increasing vibrational perturbations with $\lambda_i \neq 0$. However for the problem under consideration a numerical analysis of the stability shows that monotonic perturbations lead to a crisis of the equilibrium. The boundary of the stability in this case is determined by the condition $\lambda = 0$ and, as can be seen from Eqs. (5), it depends only on the ratio P_d/P, which is called the Lewis number $L = \chi/D$. We note that the dependence of the boundary of the monotonic instability on the Lewis number, which is absent in ordinary thermal-concentration convection, is connected with the presence of concentration sources of heat. The majority of the calculations were carried out for L in the interval $0 < L < 4$, corresponding to typical values of the Lewis number for gas mixtures.

Figure 1a and b represents examples of families of curves of neutral stability R(k) for $L = 0.5$ and $L = 2.0$ for various values of the diffusion Rayleigh number R_d. Positive R_d correspond to the case when the active component is lighter, and negative R_d denote that the heat release is due to the heavy component of the mixture. An increase in $|R_d|$ in the region $R_d < 0$ corresponds to an increase in the density of the medium, and for $R_d > 0$ it corresponds to its decrease near the lower boundary of the layer. Thus, for all Lewis numbers for $R_d < 0$ the inhomogeneities in concentration show a stabilizing action on the convective stability, and for $R_d > 0$ they show a destabilizing action. For $R_d = 0$, the variations in density are connected only with the temperature gradients. The corresponding neutral curves ($R_d = 0$) in Fig. 1 determine the threshold of convection in the case of a thermal mechanism of instability, complicated by diffusion redistribution of the heat sources.

For sufficiently large values of $R_d > 0$ the equilibrium of the mixture can prove to be unstable also in the absence of internal heat release (isothermal mixture $R = 0$). The minimum critical value R_{d*} for a pure concentration problem is determined from the equation

$$R_{d*}L = 1707.762. \tag{7}$$

(The dependence of R_{d*} on the Lewis number is connected with the choice of the units of measurement of the variables, convenient for analysis of the results in the general case.)

The neutral curves R(k) have minima $R_*(k_*)$. For $R_d > R_{d*}$ they lie in the region of negative R, which in the assumed model of the medium correspond to absorption of heat (these parts of the neutral curves on Fig. 1 are denoted by dashes).

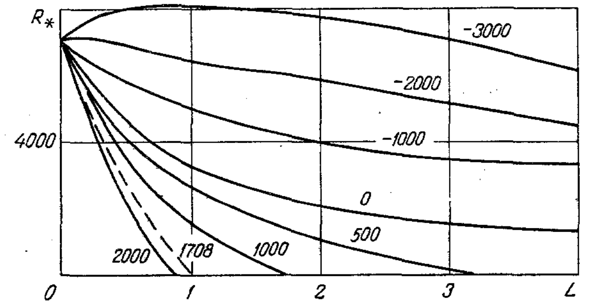

Fig. 2. Dependence of minimum critical value of Rayleigh number R_* on L for various R_d.

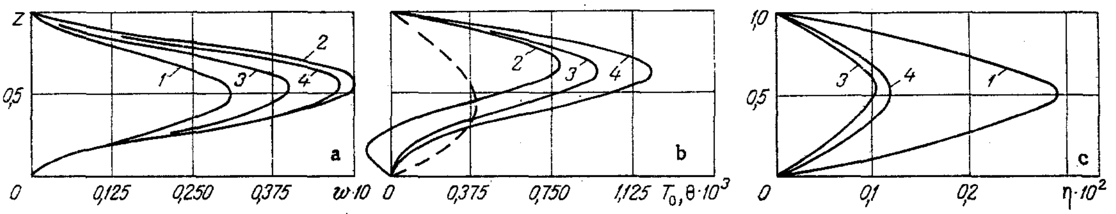

Fig. 3. Graphs of the amplitudes of the neutral perturbations of velocity (a), of the temperature (b), and of concentration (c). 1) L = 2.0; R = 0; R_d = 900; k = 3.0; 2) 0; 7040; 0; 3.5; 3) 0.5; 4590; 0; 2.8; 4) 0.5; 8000; −3000; 2.8.

For $L \lesssim 1$ the wave numbers of the critical perturbations k_* with increase in R_d practically do not vary and have the value $k_* \approx 3$. For Lewis numbers L > 1 the minimum on the neutral curves with increase in R_d is shifted in the direction of short-wave perturbations, where the value of the perturbation depends on the value of L.

Analysis of the stability of binary mixtures can conveniently be carried out on diagrams showing the dependence of the minimum value of one of the Rayleigh numbers on the remaining parameters of the problem. Such a stability diagram in (R_*, L) coordinates is shown in Fig. 2. The zones of instability are found over the $R_*(L)$ curves. The reciprocal arrangement of the stability lines on the (R_*, L) plane is determined by the diffusion Rayleigh number R_d. As has already been noted, with increasing $|R_d|$ for $R_d < 0$ the stability of the mixture increases, and for $R_d > 0$ it decreases. For $R_d = 0$ the concentration stratification is absent and the $R_*(L)$ describes the formation of an instability in the layer with interior heat sources, the inhomogeneities in the distribution of which can be equalized owing to the diffusion.

For positive values of the diffusion Rayleigh number, the characteristics $R_*(L)$ intersect the axis $R_* = 0$ at points determining the instability, caused by concentration gradients. The coordinates of these points are determined, evidently, by Eq. (7). For $R_d = 1708$ (the dashed curve) $R_* = 0$ for L = 1.

All the curves of stability $R_*(L)$ begin at L = 0 at the same point $R_* = 7025$, which determines the equilibrium crisis in the layer with inhomogeneous interior heat release. From the figure we see that for $R_d > -2000$ the fluctuations of density of the interior sources for all L decrease the stability. The speed of decrease of the critical Rayleigh number decreases with increasing Lewis number. For values $R_d < -2000$ the stability lines represent nonmonotonic curves: for $L \lesssim 1$ ($\chi < D$) R_* increases with increasing L and for L > 1 it decreases. Thus, depending on the values of the parameters R_d and L the migration of internal heat sources can affect the convective stability of equilibrium in various ways.

Together with the calculation of the threshold Rayleigh numbers for investigation of convective instability we are also interested in determining the form of the critical perturbations. For small supercriticality the generated convective motion should be similar in form to perturbations responsible for the equilibrium crisis. For finding them it is necessary, besides the eigenvalues, to seek the eigenfunctions w, θ, and η of the problem (5), (6). These functions were constructed in the form of a linear combination of the corresponding four particular solutions. The coefficients of the expansion were determined from a solution of the homogeneous system of algebraic equations for the condition of the vanishing of its determinant. One of the four coefficients is arbitrary and gives the normalization of the perturbations. In the case of a monotonic instability ($\lambda = \lambda_r$) the eigenfunctions prove to be real.

Figure 3 represents the amplitudes of the neutral perturbations of velocity (a), of temperature (b), and of concentration (c) for characteristic values of the parameters. The amplitudes found enable us to construct isotherms, lines of constant concentration and distribution of the vertical component of velocity for a convective cell. For the concentration analog of the Rayleigh problem ($R = 0$, curve 1) the distributions w and η, just as was to be expected, are symmetric with respect to the middle layer. The curves 2 refer to the other limiting case – pure thermal instability ($L = 0$, $R_d = 0$) with equilibrium profile of temperature T_0, shown in Fig. 3b by the dashed line. From the form of the distributions w and θ ($\eta = 0$) it follows that the generated convective motion is realized practically in the entire layer (penetrating convection); in this case the temperature perturbations are mainly localized in the zone of unstable thermal stratification. It is interesting to note that the amplitude of the temperature perturbation near the lower boundary of the layer changes sign.

A comparison of the curves 2 and 3 ($L = 0.5$, $R_d = 0$) shows that diffusion of the interior sources changes not only the critical Rayleigh number, but also the form of the neutral perturbations.

An example of the graphs of the perturbation amplitudes in the general case of a thermal concentration convection is represented by the curves 4. An analysis of the eigenfunctions of the problem obtained for various values of the parameters shows that the critical perturbations of velocity and of concentration exist in the entire layer; the region of temperature perturbations is broadened with an increase in the Lewis number.

In conclusion we note that the problem of the stability of equilibrium of a horizontal layer of a heat-releasing binary mixture with maximum concentration of active component on the upper boundary does not reduce to the problem considered above and requires additional investigation.

NOTATION

v	is the velocity;
T	is the temperature measured from the temperature of the boundary;
C and $C^{(0)}$	are the concentration of the heat-releasing component and its value on the lower boundary;
v_0, T_0, and C_0	are the equilibrium values of the variables of the problem;
p	is the convective contribution to the pressure;
t	is the time;
d	is the thickness of the layer;
x, y, and z	are the Cartesian coordinates;
γ	is the unit vector directed vertically upward;
β_1 and β_2	are the coefficients determining the dependence of the density on the temperature and concentration;
Q	is the specific power of heat release;
c_p	is the heat capacity at constant pressure;
χ	is the coefficient of thermal diffusivity;
γ	is the kinematic viscosity;
D	is the diffusion coefficient;
R	is the Rayleigh number;
R_d	is the diffusion Rayleigh number;
P	is the Prandtl number;
P_d	is the Schmidt number;
w, θ, and η	are the amplitudes of normal perturbations of velocity, temperature, and concentration;
λ	is the decrement of perturbations;
k_1 and k_2	are the wave numbers characterizing the periodicity of the perturbations along the x and y axes; $k^2 = k_1^2 + k_2^2$.

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DESIGN OF SUBMERGED TURBULENT JETS OF GASES OF DIFFERENT DENSITIES

V. A. Golubev

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We present results of a theoretical and experimental investigation of submerged gas jets in the range of density variations ($\rho_j/\rho_e = 0.05$-10).

A large number of studies [1-9] have been devoted to an investigation of the features of the propagation of submerged jets. Below, an attempt is made to generalize the available experimental data [2, 5, 8] for jets of various densities and to calculate certain characteristics of such jets.

For construction of the graphs (Fig. 1) we assumed the jets to be a point source placed at the pole with initial momentum equal to k_j ($k_j = \rho_j u_j^2 F_j$). The distance to the pole x_p was found from the construction of the profiles ρu^2, u, and ΔT or c at various distances from the nozzle at the $x_1 - r$ plane following the drawing of the straight lines passing through the points at which the velocity head, the velocity, and the excess temperature or concentration at each cross section of the jet attained half of their maximum (on the jet axis) value, i.e., we constructed the straight lines $r^q_{0.5}$, $r^u_{0.5}$, and $r^T_{0.5}$, originating from a single point – the pole [5].

From Fig. 1 we see that the width of the profiles ρu^2, u, and ΔT increase with decreasing density of the jet and dimensionless profiles of the excess temperatures $\Delta T/\Delta T_m$ and the concentrations c/c_m coincide [5]. In this case with an accuracy that is acceptable in practice the velocity distribution over transverse cross sections of the indicated jets is described by the theoretical profile

$$\frac{u}{u_m} = \left[1 - \left(0.44\,\frac{r/x_1}{r^u_{0.5}/x_1}\right)^{3/2}\right]^2, \tag{1}$$

and the distribution of the excess temperatures or concentrations is described by a profile which can be written as

$$\frac{\Delta T}{\Delta T_m} = \frac{c}{c_m} = \left[1 - \left(0.44\,\frac{r/x_1}{r^T_{0.5}/x_1}\right)^{3/2}\right]^2. \tag{2}$$

In Eqs. (1) and (2) the ratios of the half-maximum values of the transverse coordinates, the velocity and the temperature, $r^u_{0.5}$ and $r^T_{0.5}$ to their limiting values $r^u_{\lim}$ and $r^T_{\lim}$ are the same for all jets:

$$\frac{r^u_{0.5}}{r^u_{\lim}} = 0.44, \quad \frac{r^T_{0.5}}{r^T_{\lim}} = 0.44. \tag{3}$$

The change in the coefficient of the half-width of the jet with respect to the velocity $C^u_{0.5} = r^u_{0.5}/x_1$ and the temperature $C^T_{0.5} = r^T_{0.5}/x_1$ as a function of the relative density of the jet ρ_j/ρ_e are shown in Fig. 2. In this figure we show the variation of the coefficient of the half-width of the jet with respect to the velocity head $C^q_{0.5} = r^q_{0.5}/x_1$.

The distribution of $\rho u^2/\rho_m u_m^2$ in cross sections of the isothermal jet of air $\rho_j/\rho_e = 1.0$ (Fig. 1) corresponds to the theoretical profile obtained from Eq. (1):

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